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# Authors' reply ${ }^{2}$ 

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In our letter to the editor recently published in this journal [1], we studied a system consisting of two beam sections separated by a distributed mass regarding its natural frequencies. Unfortunately, the paper contains a few errors as some researchers whose comments were very useful have indicated [2,3,4]. The most important and conceptual one among these errors is, no doubt, that the terms representing the effect of shear force on the moment balance of distributed mass in Eq. (10) of Ref. [1] were omitted somehow. In a physically symmetric two-part beammass system it is obvious that this mistake would not affect the results because equal shear forces eliminate each other in Eq. (10). However, for a general case, the lack of these terms could lead to erroneous results. In this regard, some of the figures in Ref. [1] have been replotted after Eq. (10) was corrected. Before these new results and graphics are evaluated comparing with the existing ones in Ref. [1], two noteworthy issues will be explained in the following. The first of them is a typing mistake which appears in the definition of $\varphi$. In Eq. (24) of Ref. [1], $\varphi$ must be equal to $\rho_{2} A_{2} / \rho_{1} A_{1}$ just the inverse of that given there. Since this true definition of $\varphi$ was used in all subsequent equations, this mistake has no effect on the numerical results. In fact, the authors commenting our paper seem to realize this error, and take the parameter $\varphi$ the same as it must be according to the above definition [2].

The second issue that needs to be explained, which leads to a confusion and misinterpretation of results is that one does not draw attention to the use different $\mu$ 's in the frequency determinant (Eq. (33)), and the non-dimensional elastic curve formulas given by Eqs. (39)-(49). The elastic deflection formula (Eq. (49)) was derived considering the total weight of the system including that of the distributed mass, because it would be more reasonable, while the definitions given by Eqs. (27)-(29) based on Eq. (26) was employed in the frequency determinant. In other words, the total mass of system was defined as

$$
M_{t}=\rho_{1} A_{1}+\rho_{2} A_{2}+M
$$

[^0]and Eq. (26) was replaced with this new definition. Consequently, the mass ratios $\mu, \mu_{1}$, and $\mu_{2}$ gained new and slightly different meanings although their definitions were kept same. The need for changing the definition of total mass became necessary in the middle stage of this work. Although this modification had been reflected in the frequency determinants of both models, i.e., of the TPBMM and the BCMM, the expression of Eq. (33) were left, unfortunately, unchanged. However, if a new mass ratio, say $\mu^{*}$, is defined as
\[

$$
\begin{equation*}
\mu^{*}=\frac{\mu}{1-\mu} \tag{27'}
\end{equation*}
$$

\]



Fig. 4. Variation of the first frequency according to the BCMM and TPCMM with respect to $\eta$ and $\eta_{1}$, respectively. BCMM, --- TPBMM ( $\eta_{3}=0.005$ ), ---- TPBMM $\left(\eta_{3}=0.02\right),-----$ TPBMM $\left(\eta_{3}=0.05\right)$.


Fig. 5. Variation of the second frequency according to the BCMM and TPCMM with respect to $\eta$ and $\eta_{1}$, respectively.

- BCMM, ----- TPBMM $\left(\eta_{3}=0.005\right)$, --- TPBMM $\left(\eta_{3}=0.02\right),----$ TPBMM $\left(\eta_{3}=0.05\right)$.


Fig. 6. Variation of the third frequency according to the BCMM and TPCMM with respect to $\eta$ and $\eta_{1}$, respectively. BCMM, --- TPBMM $\left(\eta_{3}=0.005\right),---$ TPBMM $\left(\eta_{3}=0.02\right),----\operatorname{TPBMM}\left(\eta_{3}=0.05\right)$.


Fig. 8. Variation of the second frequency according to the BCMM and TPCMM with respect to $\mu$. ---$\operatorname{TPBMM}\left(\eta_{1}=0.48\right),-\operatorname{BCMM}(\eta=0.48),---\operatorname{BCMM}(\eta=0.50)$.
and used in the elements of the frequency determinant given by Eq. (33) instead of $\mu$, all plots in the paper will remain same except for the omitted shear force effect.

In what follows the corrected plots will be presented along with the ones in Ref. [1] for comparison. The same numbers as in Ref. [1] were used to enumerate the figures given here. To distinguish the subplots of a figure, including the previous, incorrect graphics, and the replotted, corrected ones, they were labelled with the letters (a) and (b), respectively. In other words, the subfigures labelled with (a) are the figures in Ref. [1] having the same figure number.

It is obvious that the curves associated with the BCMM do not alter in all the figures because moment balance is meaningless in this model. However, it is easily observed in Figs. 4(b), 5(b) and 6(b) that the curves obtained from the TPBMM have shifted down to the right in a noticeable manner especially at their peak points. While the true and incorrect frequency values are not listed numerically here, it is found out that the latter contain a relative error of $+3 \%$. The corrected


Fig. 10. Second frequency versus $\mu$ in case the mass is not located on a node.

(a)

(b)
$\eta_{1}$

Fig. 12. Variation of the first frequency according to the TPBMM with respect to $\mu$ for different values of $\varphi$. ----$\varphi=0.5,-\varphi=1,----\varphi=1.5$.


Fig. 13. Variation of the second frequency according to the TPBMM with respect to $\mu$ for different values of $\varphi .----$ $\varphi=0.5,-\varphi=1,----\varphi=1.5$.


Fig. 14. Variation of the second frequency according to the TPBMM with respect to $\eta_{1}$ for different values of $\psi$. -$\psi=0,----\psi=0.10,---\psi=0.16,---\psi=0.41$.
curves for the second and third frequencies are shown in Figs. 5(b) and 6(b). The same statement as in the case of first frequency is also valid for these frequencies.

Since the shear force terms in Eqs. (10) do not affect the results in the case of the mass located at the midpoint of the system with physically equivalent beam parts, Fig. 7 does not need to be replotted. Similarly, Fig. 9 was not reproduced due to the same reason. These two figures belong to the first and third frequencies and in the first and third modes, the mass lies parallel to the axis connecting supports. Fig. 8 had to be replotted because the mass is positioned at the modal point of second mode. As is shown in Fig. 8(b) the second frequency curve of the TPBMM coincides with the one of the BCMM.

When the mass is located for from a modal point the frequency values change and decrease relative to the incorrect ones as expected, Fig. 10(b). Fig. 10(c) show the incorrect and corrected frequency curves together.


Fig. 15. Variation of the third frequency according to the TPBMM with respect to $\eta_{1}$ for different values of $\psi$. -$\psi=0,----\psi=0.10,---\psi=0.16,---\psi=0.41$.

Fig. 11 was not replotted because it shows the variation of the first frequency according to the BCMM and the approximate method developed in Ref. [1].

Figs. 12(b) and 13(b) show the corrected curves of first and second frequencies for different $\varphi$ values. Compared with Figs. 12(a) and 13(a) (corresponding to Figs. 12 and 13 in Ref. [1]), it is seen that the curves move downward, implying a decrease in frequencies. The order of error for the first two frequencies is about $+2 \%$.

For the third frequency, there exists an error of $+5 \%$ maximum compared with the true values, although its graphics is not given here.

In Figs. 14(b) and 15(b), the frequency curves corresponding to different $\psi$ values shift downward, which indicate a decrease in frequencies in an order of $-1 \%$ compared with the incorrect values.

In the figure captions in Figs. 7-9 in Ref. [1] it is written that $\eta_{1}=0.5$ in the parentheses following the abbreviation TPBMM, while it must be $\eta_{1}=0.48$, instead. Also, in Fig. 11, the curve with solid line is attributed to the TPBMM as it is associated with the BCMM.

## References

[1] O. Kopmaz, S. Telli, On the eigenfrequencies of a two-part beam-mass system, Journal of Sound and Vibration 252 (2) (2002) 370-384.
[2] S. Naguleswaran, Comments on "On the eigenfrequencies of a two-part beam-mass system", Journal of Sound and Vibration 265 (4) (2003) 897-898, this issue.
[3] J. Banerjee, A.J. Sobey, Further investigation into the eigenfrequencies of a two-part beam-mass system, Journal of Sound and Vibration 265 (4) (2003) 899-910, this issue.
[4] S. Ilanko, Comments on "On the eigenfrequencies of a two-part beam-mass system", Journal of Sound and Vibration 265 (4) (2003) 909-910, this issue.


[^0]:    ${ }^{2}$ Reply to doi: $10.1016 / \mathrm{S} 0022-460 \mathrm{X}(02) 01670-6,10.1016 / \mathrm{S} 0022-460 \mathrm{X}(02) 01621-8,10.1016 / \mathrm{S} 0022-460 \mathrm{X}(02) 01622-\mathrm{X}$. *Corresponding author. Tel.: +90-224-442-81-76; fax: +90-224-442-91-96.
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